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UIMO
Unified International Mathematics Olympiad

UNIFIED INTERNATIONAL MATHEMATICS OLYMPIAD (UPDATED)

CLASS - 8

Question Paper Code : 4P114

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B	C	A	D	A	B	B	D	B	D

EXPLANATIONS

MATHEMATICS - 1

01. (A) Given $(5x + 4)^2 = 9(2x - 1)^2$

$25x^2 + 40x + 16 = 9(4x^2 - 4x + 1)$

$0 = 36x^2 - 25x^2 - 36x - 40x + 9 - 16$

$11x^2 - 76x - 7 = 0$

$11x^2 - 77x + x - 7 = 0$

$11x(x - 7) + 1(x - 7) = 0$

$\therefore x = 7$ (or) $x = \frac{-1}{11}$ is rejected because length is never negative

$\therefore 2x - 1 = 2(7) - 1 = 14 - 1 = 13$ m

02. (B) Volume of cuboid = Volume of cylinder

$\Rightarrow lbh = \pi r^2 h$

$\Rightarrow r^2 = \frac{44 \times 30 \times 15 \times 7}{22 \times 28}$

r = 15 cm

Hence radius of the cylinder is equal to 15 cm.

03. (B) Given $x^4 + \frac{1}{x^4} = 727$

Adding '2' on both sides

$$\left(x^2\right)^2 + \frac{1}{(x^2)^2} + 2 = 727 + 2$$

$$\left(x^2 + \frac{1}{x^2}\right)^2 = 729$$

$$x^2 + \frac{1}{x^2} = \sqrt{729} = 27$$

Subtracting '2' on both sides

$$x^2 + \frac{1}{x^2} - 2 = 27 - 2$$

$$\left(x - \frac{1}{x}\right)^2 = 25$$

$$\left(x - \frac{1}{x}\right) = \sqrt{25} = 5$$

04. (C) Base area of a cylinder = 38.5 cm^2

$\Rightarrow \pi r^2 = 38.5\ cm^2$

$\Rightarrow r = \frac{7}{2}$ cm

Lateral surface area = 176 cm^2

$\Rightarrow 2\pi rh = 176\ cm^2$

$\Rightarrow h = 8$ cm

$\therefore$ Volume $= \pi r^2 h = \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 8$

$= 308\ cm^3$

Hence volume of the cylinder

$= 308\ cm^3$

05. (D) Volume of each sheet = Volume of

$$\frac{500\,\text{sheet}}{500}$$

$$= \frac{30 \times 20 \times 5\,cm^3}{500}$$

$= 6\ cm^3$

06. (A) $\frac{xy \times yz}{xz} = \frac{8.4 \times 14.1}{11.04}$

$y^2 = 12.25$

$y = \sqrt{12.25} = 3.5$

$xyz = (xz)y = 11.04 \times 3.5 = 38.64$

07. (D) $\left(\frac{x+4}{x}\right)$

$$\text{LHS} = \left(\frac{x+1}{x}\right)\left(\frac{x+1+1}{x+1}\right)\left(\frac{x+2+1}{x+2}\right)\left(\frac{x+3+1}{x+3}\right)$$

$$= \frac{\cancel{(x+1)}}{x}\frac{\cancel{(x+2)}}{\cancel{(x+1)}}\frac{\cancel{(x+3)}}{\cancel{(x+2)}}\frac{\cancel{(x+4)}}{\cancel{(x+3)}}$$

$$= \frac{(x+4)}{x}$$

08. (D) Area = side × side

$$= \frac{9}{16}x^2 + \frac{25}{36}y^2 - \frac{5}{4}xy \text{ sq. units}$$

$$= \left(\frac{3}{4}x\right)^2 + \left(\frac{5}{6}y\right)^2 - 2 \times \frac{3}{4}x \times \frac{5}{6}y$$

$$= \left(\frac{3}{4}x - \frac{5}{6}y\right)^2 \text{ sq. units}$$

$$= \left(\frac{3}{4}x - \frac{5}{6}y\right)\left(\frac{3}{4}x - \frac{5}{6}y\right) \text{ sq. units}$$

$\therefore$ Side $= \left(\frac{3}{4}x - \frac{5}{6}y\right)$ units

09. (C) $\left(8^{-25} - 8^{-26}\right) = \frac{1}{8^{25}} - \frac{1}{8^{26}}$

$$= \frac{(8-1)}{8^{26}} = 7 \times 8^{-26}$$

10. (B) It is in direct proportion

$\therefore$ x_1 = length of the pole = 5 m 60 cm

y_1 = length of the shadow = 3 m 20 cm

x_2 = ?

y_2 = length of the shadow = 6 cm

$\therefore$ $\frac{x_1}{y_1} = \frac{x_2}{y_2}$

$$\frac{5\text{ m }60\text{ cm}}{3\text{ m }20\text{ cm}} = \frac{x_2}{6\text{m}}$$

$$\Rightarrow \quad x_2 = \frac{5.6\text{ m}}{3.2\text{ m}} \times 6\text{ m}$$

$$= \frac{\cancel{56\text{ m}}^{7}}{\cancel{32\text{ m}}_{4}} \times 6\text{ m}$$

$$= \frac{42\text{ m}}{4}$$

= 10 m 50 cm

11. (A) Let the principal be ₹ p

Given $p\left(1+\frac{r}{100}\right)^n - p = CI$

$$\Rightarrow p\left(1+\frac{5}{100}\right) - p = ₹\,1324.05$$

$$p\left(\frac{21}{20}\times\frac{21}{20}\times\frac{21}{20}\right) - p = ₹\,1324.05$$

$$\frac{9261p}{8000} - p = ₹\,1324.05$$

$$\frac{9261p - 8000p}{8000} = ₹\,1324.05$$

1261p = 8000 × ₹ 1324.05

$$p = \frac{₹\,10592400}{1261} = ₹\,8400$$

$$SI = \frac{PTR}{100} = ₹\,1260$$

12. (D) $2^8 + 1 = 256 + 1 = 257$

$\therefore$ $2^{18} + 1 = (2^6)^3 + 1 = 64^3 + 1$

$257 < 7^3 < 64^3 < 64^3 + 1$

$\therefore$ No. of perfect cubes = 64 – 7 + 1 = 58

13. (B) First term

$$= \sqrt{1+1+\frac{1}{4}} = \sqrt{\frac{9}{4}} = \frac{3}{2} = 2 - \frac{1}{2}$$

Sum of first two terms = $\frac{3}{2} + \sqrt{1+\frac{1}{4}+\frac{1}{9}}$

$$= \frac{3}{2} + \sqrt{\frac{36+9+4}{36}} = \frac{3}{2} + \frac{7}{6} = \frac{9+7}{6} = \frac{\cancel{16}^{8}}{\cancel{6}_{3}}$$

$$= 3 - \frac{1}{3}$$

Sum of first three terms

$$= \frac{8}{3} + \sqrt{1+\frac{1}{9}+\frac{1}{16}} = \frac{8}{3} + \frac{13}{12}$$

$$\frac{45}{12} = \frac{15}{4} = 4 - \frac{1}{4}$$

$\therefore$ Sum of all terms = $2021 - \frac{1}{2021}$

14. (C) $2x^2 - 3y^2 = \left(\sqrt{2}\right)^2 x^2 - \left(\sqrt{3}\right)^2 y^2$

$$= \left(\sqrt{2}x\right)^2 - \left(\sqrt{3}y\right)^2$$

$$= \left(\sqrt{2}x + \sqrt{3}y\right)\left(\sqrt{2}x - \sqrt{3}y\right)$$

15. (B) Since $x + xy = 391$, then $x(1 + y) = 391$.

We note that 391 = 17 × 23

Since 17 and 23 are both prime, then if 391 is written as the product of two positive integers, it must be 1 × 391 or 17 × 23 or 23 × 17 or 391 × 1.

Matching x and $1 + y$ to these possible factors, we obtain (x, y) = (1, 390) or (17, 22) or (23, 16) or (391, 0).

Since y is a positive integer, the fourth pair is not possible.

Since $x > y$, the first two pairs are not possible.

Therefore, (x, y) = (23, 16)

$\Rightarrow x + y = 39$

16. (A) d = 6 as

$6^3 = 3^3 + 4^3 + 5^3$

17. (D) Originally, let there be x men. Less men $\Rightarrow$ more days.

$\therefore \quad (x-10) : x :: 100 : 110$

$\Rightarrow (x-10) \times 110 = x \times 100$

$\Rightarrow 10x = 1100 \Rightarrow x = 110$

18. (C) $4(x^2 + 10x + 25) - (4x^2 + 4x + 1)$

$= 3x - 15 + 180$

$4x^2 + 40x + 100 - 4x^2 - 4x - 1$

$= 3x + 165$

$36x + 99 = 3x + 165$

$33x = 66$

$x = 2$

19. (B) Let the cost price of each candel be ₹x

$\therefore$ cost price of 15 candles = ₹$15x$

selling price of 12 candle = ₹$15x$

selling price of each candle = $\frac{₹\cancel{15}^{5}x}{\cancel{12}_{4}} = \frac{₹5x}{4}$

$\therefore$ Profit = $\frac{₹5x}{4} - ₹x = \frac{₹5x - ₹4x}{4} = \frac{₹x}{4}$

Profit percentage

$$= \frac{\text{Profit}}{\text{CP}} \times 100 = \frac{\left(\frac{₹x}{4}\right)}{₹x} \times 100 = 25\%$$

(OR)

Let CP of each candle be ₹12 &

SP of each candle be ₹15

$\therefore$ Profit = ₹15 – ₹12 = ₹3

Profit percentage

$$= \frac{\text{Profit}}{\text{CP}} \times 100 = \frac{₹3}{₹12} \times 100 = 25\%$$

20. (A)

4	18670918 16	4321
83	267 249	
862	1809 1724	
8641	8518 8641	
	−123	

$4321^2 = 18670918 + 123 = 18671041$

21. (A) $2^{2028} + 2^{2027} - 2^{2026} - 2^{2025} = 2^{2025} \times 2^3 + 2^{2025} \times 2^2 - 2^{2025} \times 2 - 2^{2025} \times 1$

$= 2^{2025}(8 + 4 - 2 - 1)$

$= 9 \times 2^{2025}$

$= 3^2 \times 2^2 \times 2^{2023}$

$= 6^2 \times 2^{2023}$

22. (A)

$$\sqrt[3]{\frac{0.2\times0.2\times0.2+0.04\times0.04\times0.04}{0.2\times2\times0.2\times2\times0.2\times2+0.04\times2\times0.04\times2\times0.04\times2}}$$

$$= \sqrt[3]{\frac{\cancel{(0.2\times0.2\times0.2+0.04\times0.04\times0.04)}}{\cancel{8(0.2\times0.2\times0.2+0.04\times0.04\times0.04)}}}$$

$$= \sqrt[3]{\left(\frac{1}{2^3}\right)}$$

$$= \frac{1}{2} = 0.5$$

23. (C) Given $2\pi r = 220$ cm

$2 \times \frac{22}{7} r = 220$ cm

$r = 220 \text{ cm} \times \frac{7}{44} = 35$ cm

Volume of the cylinder

$= \pi r^2 h = \frac{22}{\cancel{7}} \times \cancel{35}^{5} \times 35 \times 63 \text{ cm}^3$

$= 2,42,550 \text{ cm}^3$

24. (C) The bacteria two hours back be 'x'

Given $x\left(1 + \frac{2.5}{100}\right)^2 = 5,37,920$

$x\left(1 + \frac{25}{1000}\right)^2 = 5,37,920$

$x \times \frac{41}{40} \times \frac{41}{40} = 5,37,920$

$x = 5,37,920 \times \frac{40}{41} \times \frac{40}{41}$

$= 5,12,000$

25. (C) $(87654322)^2 - (12345678)^2 = (87654322 - 12345678)(87654322 + 12345678)$

$= 75308644 \times 100000000$

$= 7530864400000000$

26. (C) $x + y = \frac{1}{x} + \frac{1}{y}$

$= x + y = \frac{(x+y)}{xy}$

$= xy = \frac{(x+y)}{(x+y)} = 1$

27. (C) $\text{LHS} = \frac{25}{3} \times \frac{-6}{65} \times \frac{-1}{2} = \frac{5}{13}$

28. (C) Let the smallest angle be $x°$

Given $x + 3x + x + 3x + 20° = 180°$

$8x = 160°$

$x = 20°$

$3x = 60°$ & $4x + 20° = 100°$

29. (C) Option A : $18^2 + 27^2 + 32^2 = 324 + 729 + 1024 = 2077$

Option B : $7^2 + 24^2 + 26^2 + 27^2 = 49 + 576 + 676 + 729 = 2030$

Option C : $20^2 + 28^2 + 29^2 = 400 + 784 + 841 = 2025$

Option D : $21^2 + 28^2 + 30^2 = 441 + 784 + 900 = 2125$

$\therefore$ Option 'C' is correct

30. (C) $2x^2 + \sqrt{3}x - 3$

$= 2x^2 + 2\sqrt{3}x - \sqrt{3}x - 3$

$= 2x\left(x + \sqrt{3}\right) - \sqrt{3}\left(x + \sqrt{3}\right)$

$= \left(x + \sqrt{3}\right)\left(2x - \sqrt{3}\right)$

MATHEMATICS - 2

31. (A,B,C,D)

BAg 'A' = 6 kg Bag 'B'

= 18 kgs Bag 'c' = 14 kgs.

32. (A,B,C,D)

It is an identify.

Hence it is true for all natural, whole, integers and rational numbers.

33. (B,C)

$2025^2 - 2037 = 2025^2 - 2025 - 12$

$= x^2 - x - 12$ where $x = 2025$

$= x^2 - 4x + 3x - 12$

$= x(x - 4) + 3(x - 4)$

$= (x - 4)(x + 3)$

$= (2025 - 4)(2025 + 3)$

$= 2021 \times 2028$

2021 & 2028 are the factors of $2025^2 - 2037$. (or)

$2025^2 - 2037 = 4100625 - 2037 = 4098588$

4098588 units place is '8' Hence there is a chance for 2021×2028.

34. (A,C,D)

$13^4 - 11^4 = (13^2)^2 - (11^2)^2 = (13^2 - 11^2)(13^2 + 11^2)$

$= (13 + 11)(13 - 11) \mid (169 + 121)$

$= 24 \times 2 \times 290$

$\therefore$ 2, 3 & 29 are the factors of $13^4 - 11^4$

35. (A,B,C,D)

$\frac{-3}{20} = -0.15$

$\frac{-3}{11} = -0.27$

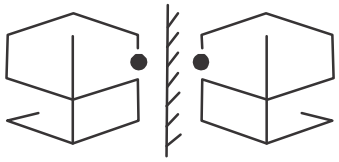
$\frac{-5}{22} = -0.22$ lies between $\frac{-3}{20}$ and $\frac{-3}{11}$

REASONING

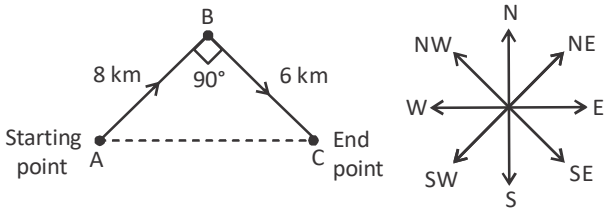
36. (B) Common in sentences 1 & 2: health, problems → common codes ho, la

Common in sentences 2 & 3: rise → common code ta

So, ta = rise.

37. (D) 

38. (B) According to the question,



Here, total distance = AB + BC

= 8 + 6 = 14 km

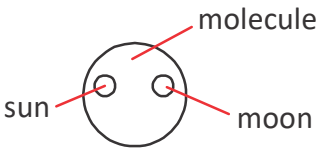
and shortest distance AC = $AB^2 + BC^2$

= $8^2 + 6^2$

= 64 + 36

= 100 = 10 km

39. (C) Sun and Moon in Molecule



40. (C) In the first fig, (6 × 4) + (5 × 2)

= 24 + 10 = 34

In the second fig, (4 × 3) + (7 × 9)

= 12 + 63 = 75

In the third figure (8 × 4) + (4 × 6)

= 24 + 12 = 56

41. (B) 

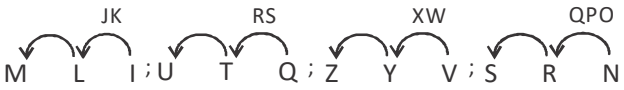
42. (C) Days in january = 31 → days remaining after jan 1 up to mar 1 : 31 (jan) + 28 (feb, 2025 is not a leap year) = 59 days.

Advance from jan 1 by 59 days → 59 mod 7 = 33 days forward.

jan 1, 2025 was wednesday; add 3 days → saturday.

43. (A) A king sits on a throne; a judge sits on a bench.

44. (D) In all other groups, there is a gap of two letters between second and third letters in the alphabetical order.



45. (A) 

CRITICAL THINKING

46. (B) We need winners' paths. From results:

Madhav beats Teja

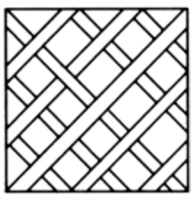
Madhav beats Varun

Madhav beats Rohan So Madhav won three matches → champion. Teja beats Arjun and Krishna but loses to Madhav → Teja reached final and lost to Madhav. Therefore final was Madhav vs Teja.

47. (B) A normal n × n magic square using the numbers 1 to n2 has row / column/ diagonal sum (magic constant)

$$M = \frac{n\left(n^2+1\right)}{2} \text{ for } n = 4$$

$$M = \frac{4\left(16+1\right)}{2} = \frac{4\times 17}{2} = 2\times 17 = 34$$

48. (D) 

49. (B) 

50. (D)